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Cubic Difference Prime Labeling of Some Snake Graphs

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Abstract: *Cubic difference prime labeling of a graph is the labeling of the vertices with {0,1,2-----,p-1} and the edges with absolute difference of the cubes of the labels of the incident vertices. The greatest common incidence number of a vertex (gcin) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits cubic difference prime labeling. Here we identify some snake graphs for cubic difference prime labeling.*

Keywords: *Graph labeling, cubic difference, greatest common incidence number, prime labeling, snake graphs.*

1. INTRODUCTION

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4]. Some basic concepts are taken from [1] and [2]. In this paper we developed the idea of cubic difference prime labeling using the concept greatest common incidence number. Here we investigated cubic difference prime labeling of some snake graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2. MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection
f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping
f_{cdpl}^{*} : E(G) → set of natural numbers N by $f_{cdpl}^{*}(uv) = |\{f(u)\}^3 - \{f(v)\}^3|$ The induced function f_{cdpl}^{*} is said to be cubic difference prime labeling, if the ***gcin*** of each vertex of degree at least 2, is 1.

Definition 2.2 A graph which admits cubic difference prime labeling is called a cubic difference prime graph.

Theorem 2.1 Triangular Snake T_n admits cubic difference prime labeling.

Proof: Let G = T_n and let v_1, v_2,----------------,v_{2n-1} are the vertices of G

Here |V(G)| = 2n-1 and |E(G)| = 3n-3

Define a function f : V → {0,1,2,3,----------------,2n-2 } by

$f(v_i) = i-1$, i = 1,2,------,2n-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{cdpl}^{*} is defined as follows

$f_{cdpl}^{*}(v_i\, v_{i+1}) = i^3-(i-1)^3$, i = 1,2,------------,2n-2

$f_{cdpl}^{*}(v_{2i-1}\, v_{2i+1}) = 24i^2 - 24i + 8$, i = 1,2,------------,n-1

Clearly f_{cdpl}^{*} is an injection.

gcin of (v_{i+1}) = gcd of {$f_{cdpl}^{*}(v_i\, v_{i+1})$, $f_{cdpl}^{*}(v_{i+1}\, v_{i+2})$ }
= gcd of { $3i^2-3i+1$, $3i^2+3i+1$}
= gcd of {$6i^2$, $3i^2-3i+1$}
= gcd of {$3i^2$, $3i^2-3i+1$}
= gcd of {3i-1 , $3i^2-3i+1$}
= gcd of {i, 3i-1}

= gcd of {i, i-1} = 1,
i = 1,2,------------,2n-3

gcin of (v_1) = gcd of {1,8}=1

gcin of (v_{2n-1}) = gcd of {$f_{cdpl}^{*}\,(v_{2n-2}\ v_{2n-1})$, $f_{cdpl}^{*}\,(v_{2n-3}\ v_{2n-1})$ }
= gcd of { $12n^2$-30n+19, $24n^2$-72n+56}
= gcd of { $12n^2$-30n+19, $12n^2$-36n+28}
= gcd of { $12n^2$-30n+19, 6n-9}
= gcd of { (6n-9)(2n-2)+1, 6n-9} = 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence T_n , admits cubic difference prime labeling. ■

Example 2.1 G = T_5

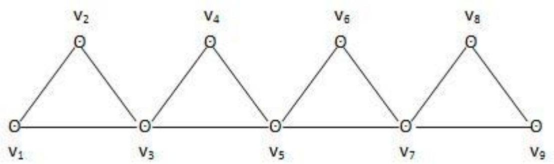

fig – 2.1

Theorem 2.2 Quadrilateral Snake Q_n admits cubic difference prime labeling, when (n+4)≢0(mod7).

Proof: Let G = Q_n and let v_1,v_2,----------------,v_{3n-2} are the vertices of G

Here |V(G)| = 3n-2 and |E(G)| = 4n-4

Define a function f : V → {0,1,2,3,----------------,3n-3 } by
f(v_i) = i-1 , i = 1,2,------,3n-2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{cdpl}^{*} is defined as follows

$f_{cdpl}^{*}\,(v_i\ v_{i+1})$ = i^3-$(i-1)^3$, i = 1,2,------------,3n-3

$f_{cdpl}^{*}\,(v_{3i-2}\ v_{3i+1})$ = $81i^2 - 81i + 27$, i = 1,2,------------,n-1

Clearly f_{sqsp}^{*} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,3n-4

gcin of (v_1) = gcd of {1,27} = 1.

gcin of (v_{3n-2}) = gcd of {$f_{cdpl}^{*}\,(v_{3n-2}\ v_{3n-3})$, $f_{cdpl}^{*}\,(v_{3n-2}\ v_{3n-5})$ }
= gcd of { $27n^2$-63n+37, $81n^2$-243n+189}
= gcd of { $27n^2$-63n+37, $27n^2$-81n+63}
= gcd of { 9n-13, $27n^2$-81n+63}
= gcd of { 9n-13, 3n-2}
= gcd of { 3n-9, 3n-2}
= gcd of { 3n-9, 7} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence Q_n , admits cube difference prime labeling. ■

Example 2.2 G = Q_4

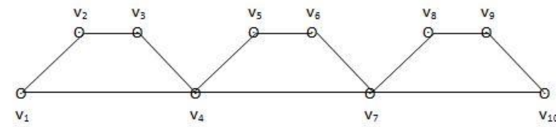

fig -2.2

Theorem 2.3 Alternate Triangular Snake A(T_n) admits cubic difference prime labeling, when n is even and the triangle starts from first vertex.

Proof: Let G =A(T_n) and let v_1,v_2,----------------,$v_{\frac{3n}{2}}$ are the vertices of G

Here |V(G)| = $\frac{3n}{2}$ and |E(G)| = 2n-1

Define a function f : V → {0,1,2,3,----------------,$\frac{3n-2}{2}$ } by
f(v_i) = i-1 , i = 1,2,------, $\frac{3n}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{cdpl}^{*} is defined as follows

$f_{cdpl}^{*}\,(v_i\ v_{i+1})$ = i^3-$(i-1)^3$, i = 1,2,------------,$\frac{3n-2}{2}$

$f_{cdpl}^{*}\,(v_{3i-2}\ v_{3i})$ = $54i^2 - 72i + 26$, i = 1,2,-------------,$\frac{n}{2}$

Clearly f_{cdpl}^{*} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,$\frac{3n-4}{2}$

gcin of (v_1) = 1

gcin of ($v_{\frac{3n}{2}}$) = gcd of {$f_{cdpl}^{*}\,(v_{(\frac{3n}{2})}\ v_{(\frac{3n-2}{2})})$, $f_{cdpl}^{*}\,(v_{(\frac{3n}{2})}\ v_{(\frac{3n-4}{2})})$ }
= gcd of { $\frac{54n^2-108n+56}{8}$, $\frac{108n^2-288n+208}{8}$}
= gcd of {$\frac{27n^2}{4}$ - $\frac{54n}{4}$ +7, $\frac{54n^2}{4}$ - $\frac{144n}{4}$ +26}
= gcd of {$\frac{27n^2}{4}$ - $\frac{54n}{4}$ +7, $\frac{27n^2}{4}$ - $\frac{90n}{4}$ +19}
= gcd of {$9n-12$, $\frac{27n^2}{4}$ - $\frac{90n}{4}$ +19}
= gcd of {$9n-12$,(9n-12)($\frac{3n}{4}$ - $\frac{6}{4}$)+1}=1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A(T_n) , admits cubic difference prime labeling. ■

Example 2.3 G = A(T_4)

fig -2.3

Theorem 2.4 Alternate Triangular Snake $A(T_n)$ admits cubic difference prime labeling, when n is even and the triangle starts from second vertex.

Proof: Let G =A(T_n) and let v_1, v_2,----------------,$v_{(\frac{3n-2}{2})}$ are the vertices of G

Here |V(G)| = $\frac{3n-2}{2}$ and |E(G)| = 2n-3

Define a function f : V → {0,1,2,3,----------------, $\frac{3n-4}{2}$ } by

$f(v_i)$ = i-1 , i = 1,2,------, $\frac{3n-2}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i\ v_{i+1})$ = i^3-$(i-1)^3$, i = 1,2,------------,$\frac{3n-4}{2}$

$f^*_{cdpl}(v_{3i-1}\ v_{3i+1})$ = $54i^2 - 36i + 8$, i = 1,2,------------,$\frac{n-2}{2}$

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,$\frac{3n-6}{2}$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A(T_n) , admits cubic difference prime labeling. ■

Example 2.4 G = A(T_6)

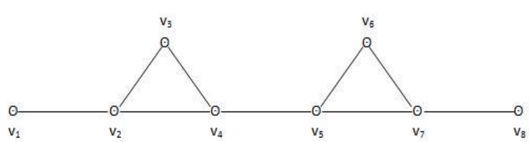

Fig – 2.4

Theorem 2.5 Alternate Triangular Snake $A(T_n)$ admits cubic difference prime labeling, when n is odd and the triangle starts from first vertex.

Proof: Let G =A(T_n) and let v_1, v_2,----------------,$v_{(\frac{3n-1}{2})}$ are the vertices of G

Here |V(G)| = $\frac{3n-1}{2}$ and |E(G)| = 2n-2

Define a function f : V → {0,1,2,3,----------------, $\frac{3n-3}{2}$ } by

$f(v_i)$ = i-1 , i = 1,2,------, $\frac{3n-1}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i\ v_{i+1})$ = i^3-$(i-1)^3$, i = 1,2,------------,$\frac{3n-3}{2}$

$f^*_{cdpl}(v_{3i-2}\ v_{3i})$ = $54i^2 - 72i + 26$, i = 1,2,-------------,$\frac{n-1}{2}$

Clearly f^*_{cdpl} is an injection.

gcin of (v_1) = 1

gcin of (v_{i+1}) = 1, i = 1,2,------------,$\frac{3n-5}{2}$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A(T_n) , admits cubic difference prime labeling. ■

Example 2.5 G = A(T_5)

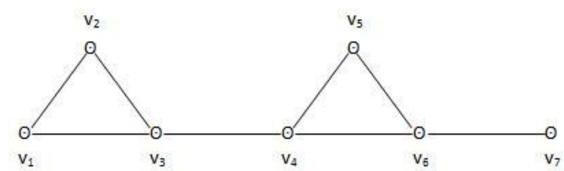

fig – 2.5

Theorem 2.6 Alternate Triangular Snake $A(T_n)$ admits cubic difference prime labeling, when n is odd and the triangle starts from second vertex.

Proof: Let G =A(T_n) and let v_1, v_2,----------------,$v_{(\frac{3n-1}{2})}$ are the vertices of G

Here |V(G)| = $\frac{3n-1}{2}$ and |E(G)| = 2n-2

Define a function f : V → {0,1,2,3,----------------, $\frac{3n-3}{2}$ } by

$f(v_i)$ = i-1 , i = 1,2,------, $\frac{3n-1}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i\ v_{i+1})$ = i^3-$(i-1)^3$, i = 1,2,------------,$\frac{3n-3}{2}$

$f^*_{cdpl}(v_{3i-1}\ v_{3i+1})$ = $54i^2 - 36i + 8$, i = 1,2,------------,$\frac{n-1}{2}$

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,$\frac{3n-5}{2}$

gcin of $(v_{(\frac{3n-1}{2})})$ = gcd of { $f^*_{cdpl}(v_{(\frac{3n-1}{2})}\ v_{(\frac{3n-3}{2})})$, $f^*_{cdpl}(v_{(\frac{3n-1}{2})}\ v_{(\frac{3n-5}{2})})$ }

= gcd of { $\frac{27n^2}{4}$ - 18n+$\frac{49}{4}$, $\frac{27n^2}{2}$ - 45n+$\frac{79}{2}$}

$= \text{gcd of } \{ \frac{27n^2}{4} - 18n + \frac{49}{4}, \frac{27n^2}{4} - 27n + \frac{109}{4} \}$
$= \text{gcd of } \{ 9n - 15, \frac{27n^2}{4} - 27n + \frac{109}{4} \}$
$= \text{gcd of } \{ 9n - 15, (9n-15)(\frac{3n-7}{4}) + 1 \}$
$= 1.$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A(T_n) , admits cubic difference prime labeling. ■

Example 2.6 G = A(T_5)

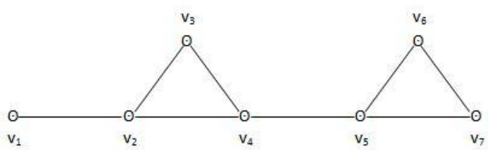

fig -2.6

Theorem 2.7 G be the graph obtained from comb graph by replacing the path edges by triangles. G is called comb triangular snake graph and is denoted by Comb(T_n). G admits cubic difference prime labeling.

Proof: Let G = Comb(T_n) and let v_1, v_2,----------------,v_{3n-1} are the vertices of G

Here |V(G)| = 3n-1 and |E(G)| = 4n-3

Define a function f : V → {0,1,2,3,----------------,3n-2 } by

$f(v_i)$ = i-1 , i = 1,2,------,3n-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i\, v_{i+1}) = i^3 - (i-1)^3$, i = 1,2,------------,2n

$f^*_{cdpl}(v_{2i}\, v_{2i+2}) = 24i^2 + 2$, i = 1,2,------------,n-1

$f^*_{cdpl}(v_{2i+2}\, v_{2n+i+1}) = (2n+i)^3 - (2i+1)^3$, i = 1,2,------------,n-1

Clearly f^*_{cdpl} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,2n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence comb(T_n) , admits cubic difference prime labeling. ■

Example 2.7 G = Comb(T_5)

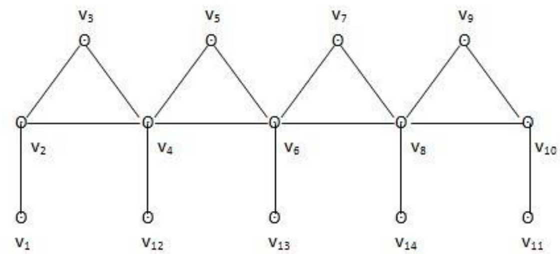

fig – 2.7

Theorem 2.8 Alternate Quadrilateral Triangular Snake A{$(QT)_n$} admits cube difference prime labeling, when n is odd

Proof: Let G = A{$(QT)_n$} and let v_1, v_2,----------------,$v_{(\frac{5n-3}{2})}$ are the vertices of G

Here |V(G)| = $\frac{5n-3}{2}$ and |E(G)| = $\frac{7n-7}{2}$

Define a function f : V → {0,1,2,3,----------------, $\frac{5n-5}{2}$ } by

$f(v_i)$ = i-1 , i = 1,2,------, $\frac{5n-3}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i\, v_{i+1}) = 3i^2 - 3i + 1$, i = 1,2,------------,$\frac{5n-5}{2}$

$f^*_{cdpl}(v_{5i-4}\, v_{5i-1}) = 225i^2 - 315i + 117$, i = 1,2,------------,$\frac{n-1}{2}$

$f^*_{cdpl}(v_{5i-1}\, v_{5i+1}) = 150i^2 - 60i + 8$, i = 1,2,------------,$\frac{n-1}{2}$

Clearly f^*_{cdpl} is an injection.

gcin of (v_1) = 1

gcin of (v_{i+1}) = 1, i = 1,2,------------,$\frac{5n-7}{2}$

gcin of ($v_{(\frac{5n-3}{2})}$) = gcd of { $f^*_{cdpl}(v_{(\frac{5n-3}{2})}\, v_{(\frac{5n-5}{2})})$, $f^*_{cdpl}(v_{(\frac{5n-3}{2})}\, v_{(\frac{5n-7}{2})})$ }

= gcd of { $\frac{150n^2-360n+218}{8}$, $\frac{300n^2-840n+604}{8}$, }= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence A{$(QT)_n$}, admits cube difference prime labeling. ■

Theorem 2.9 Alternate Triangular Quadrilateral Snake A{$(TQ)_n$} admits cube difference prime labeling, when n is even.

Proof: Let G = A{$(TQ)_n$} and let v_1, v_2,---------------,$v_{(\frac{5n-4}{2})}$ are the vertices of G

Here $|V(G)| = \frac{5n-4}{2}$ and $|E(G)| = \frac{7n-8}{2}$

Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------}, \frac{5n-6}{2}\}$ by

$f(v_i) = i-1$, $i = 1,2,\text{------}, \frac{5n-4}{2}$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_i\ v_{i+1}) = 3i^2 - 3i + 1$, $\quad i = 1,2,\text{------------}, \frac{5n-6}{2}$

$f^*_{cdpl}(v_{5i-4}\ v_{5i-2}) = 150i^2 - 240i + 98$, $\quad i = 1,2,\text{------------}, \frac{n}{2}$

$f^*_{cdpl}(v_{5i-2}\ v_{5i+1}) = 225i^2 - 135i + 27$, $\quad i = 1,2,\text{------------}, \frac{n-2}{2}$

Clearly f^*_{cdpl} is an injection.

gcin of (v_1) $= 1$

gcin of $(v_{i+1}) = 1$, $\quad i = 1,2,\text{------------}, \frac{5n-8}{2}$

gcin of $(v_{(\frac{5n-3}{2})})$ = gcd of $\{f^*_{cdpl}(v_{(\frac{5n-6}{2})}\ v_{(\frac{5n-4}{2})})$, $f^*_{cdpl}(v_{(\frac{5n-8}{2})}\ v_{(\frac{5n-4}{2})})\}$

= gcd of $\{\frac{150n^2-420n+246}{8}$, $\frac{300n^2-960n+784}{8}$, $\} = 1$.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $A\{(TQ)_n\}$, admits cube difference prime labeling. ■

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